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IM. STANISŁAWA STASZICA W KRAKOWIE

Strength of Materials

Design of structural joints

Example of joints, cross-sectional forces and stresses, idealization of connector work, strength criteria for shear and contact pressure, fillet welds

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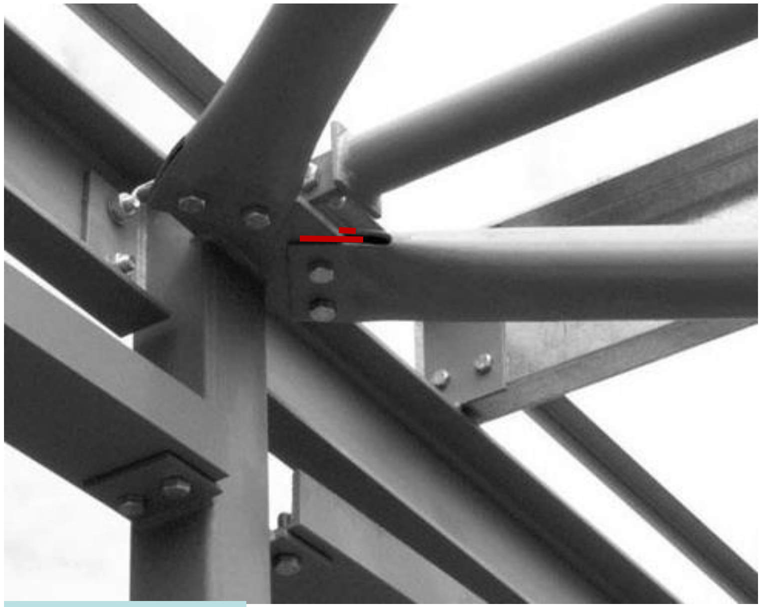
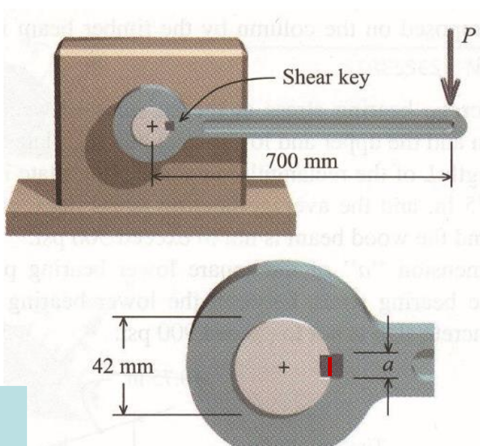
Examples of joints



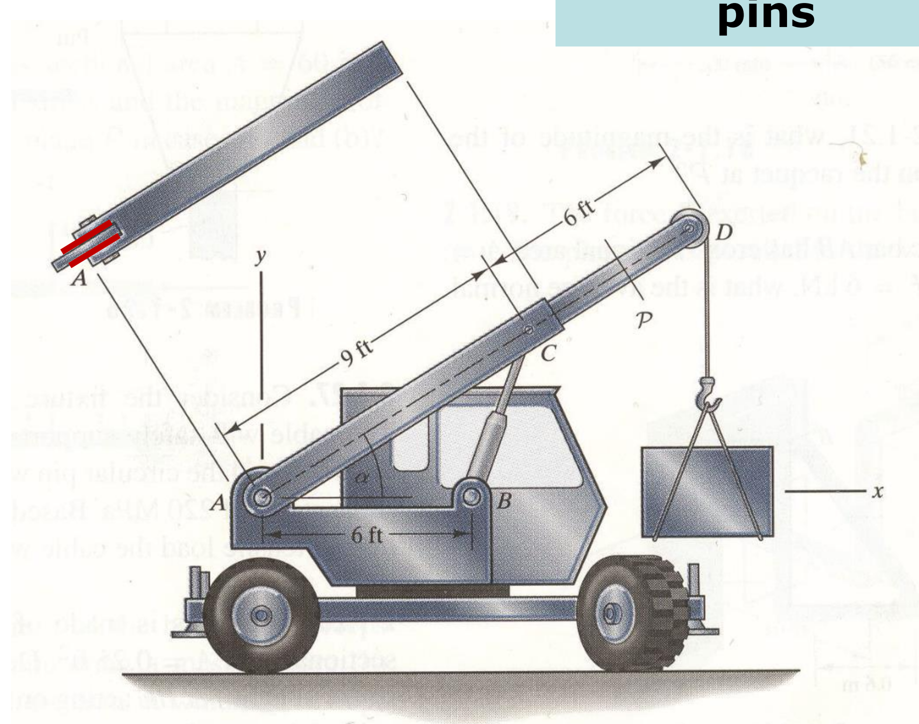
bolts



Philpot T.A.: Mechanics of materials

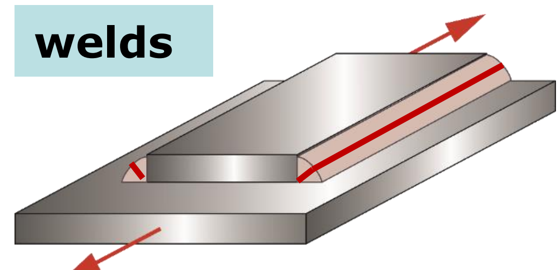
keys

pins

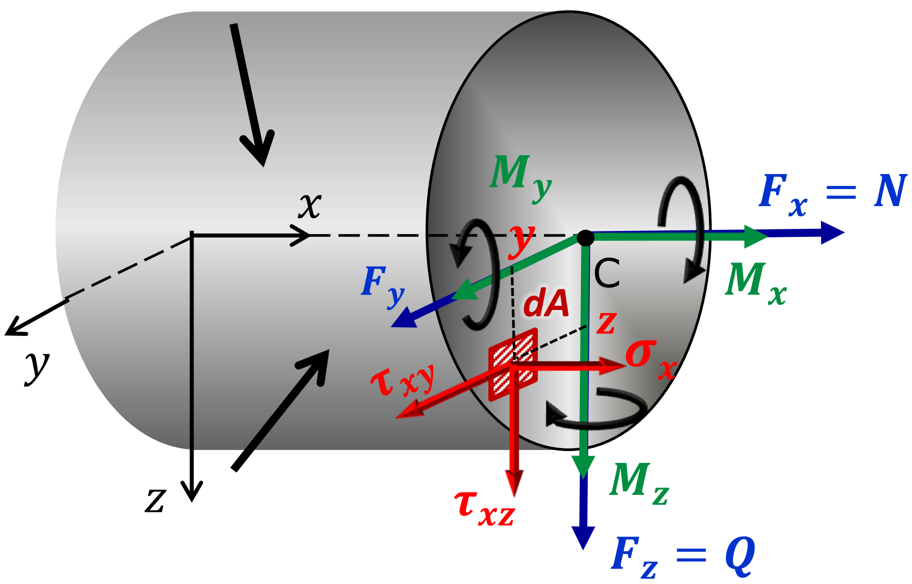


Bedford A.: Mechanics of materials

welds



Cross-sectional forces and stresses



$$N = \int_A \sigma_x \, dA$$ - tension/ compression

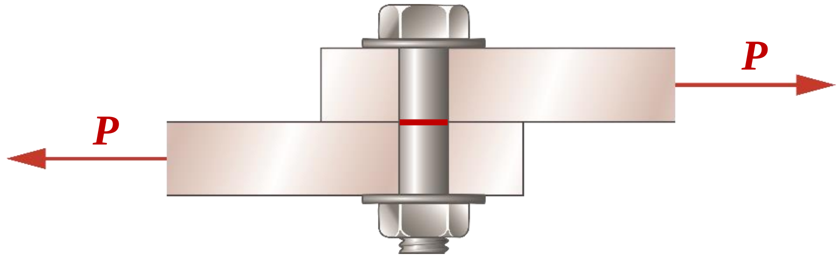
$$F_y = \int_A \tau_{xy} \, dA$$
$$F_z = \int_A \tau_{xz} \, dA$$
- shear

$$T = \sqrt{F_y^2 + F_z^2}$$

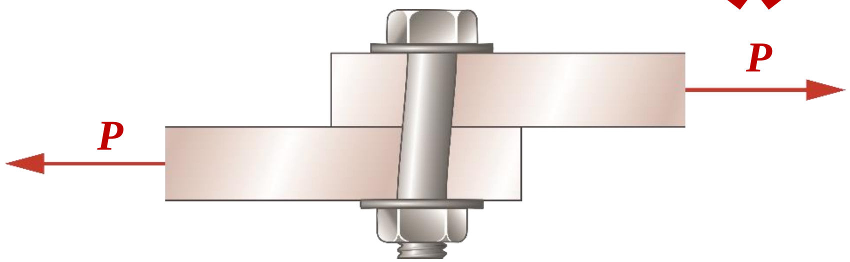
$$M_x = \int_A (\tau_{xz}\, y - \tau_{xy}\, z) \, dA$$ - torsion

$$M_y = \int_A \sigma_x \, z \, dA$$
$$M_z = -\int_A \sigma_x \, y \, dA$$
- bending

Direct shear
Idealization

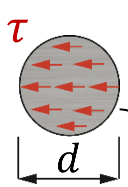


Connector bending



Approximate distribution of shear stresses in the connector's cross-section

$$\tau = \frac{P}{A_t}$$



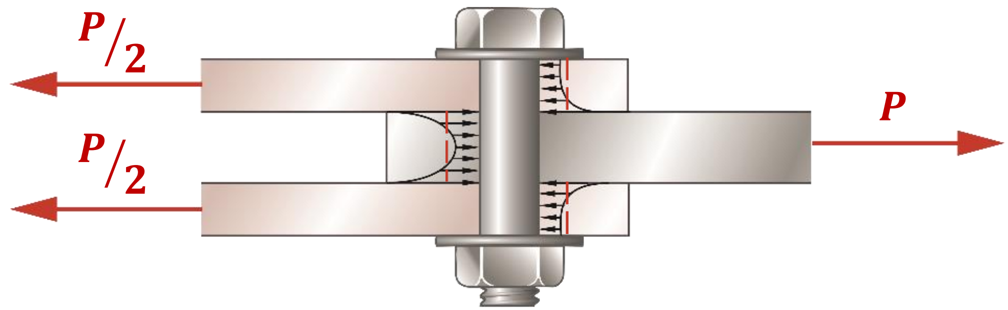
$$A_t = \frac{\pi d^2}{4}$$

Connections with bolts, rivets, and pins are designed with the following assumptions:

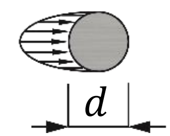
- Bending is omitted because stresses generated by them are significantly lower than those induced by shear.
- The shear force acting in the bolt's cross-section caused uniform stress distribution, which approximates the actual.

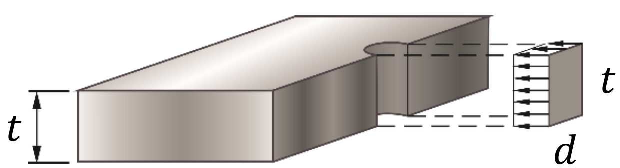
Contact pressure
Idealization

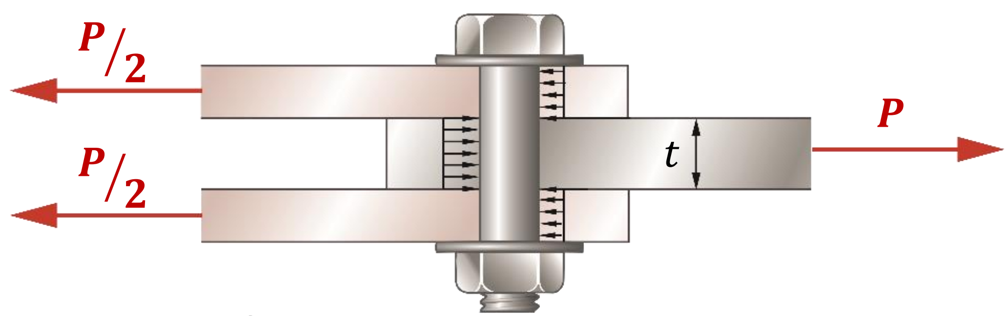
The actual stress distribution on the cylindrical contact surface between a plate and the connector's mandrel.



Approximate stress distribution on the equivalent contact surface created by the plate thickness *t* and the connector diameter *d*.



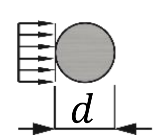




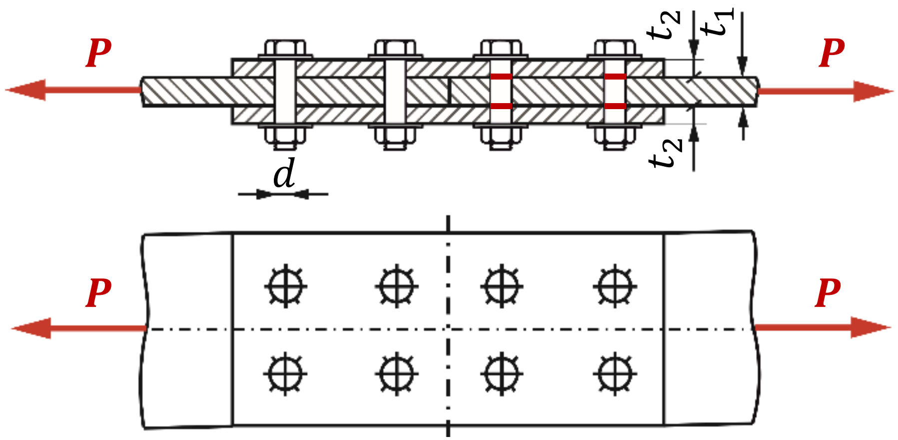
Bearing stress:

$$\sigma_d = \frac{P}{t \cdot d}$$

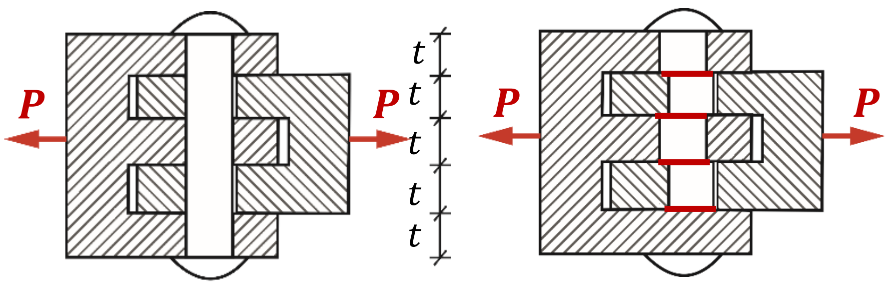
If the permissible value for this stress is exceeded, the plate becomes plasticized

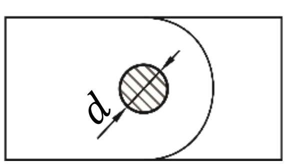


Strength criterion
Shearing of connector's mandrel



$n = 4, i = 2$





$n = 1, i = 4$

Shear stress single connectors:

$$\tau = \frac{P/n}{A_t}$$

Shear surface:

$$A_t = \frac{\pi d^2}{4} \cdot i$$

Strength criterion for shearing:

$$\frac{\mathbf{4P}}{\boldsymbol{\pi d^2 \cdot n \cdot i}} \leq \boldsymbol{\sigma_{allow.S}}$$

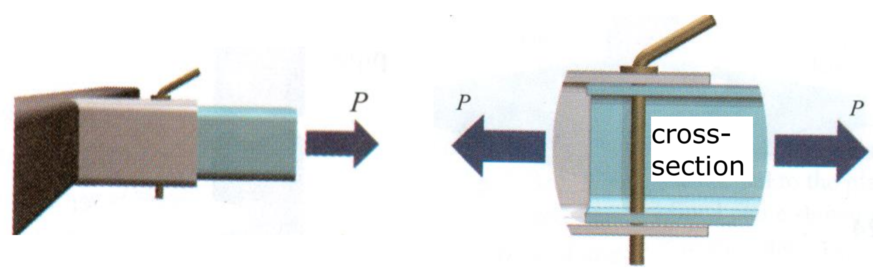
d - connector diameter

n - number of force-transmitting connectors

i - number of shear planes $\quad i = m - 1$

m - number of plates covered by the connector

$\boldsymbol{\sigma_{allow.S}}$ - allowable shear stress

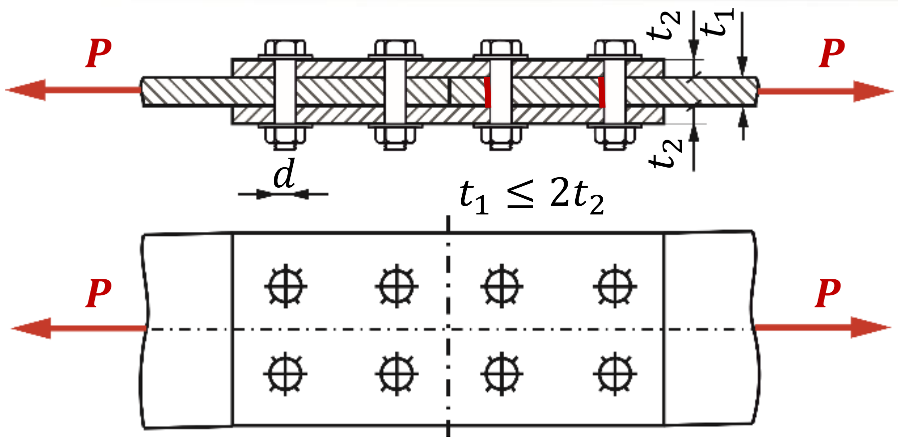


$n = 1, i = 2$

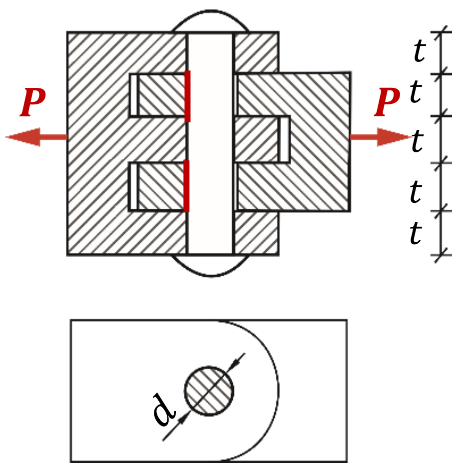
M. Niezgodziński, et. al., Zadania z wytrzymałości materiałów

Philpot T.A.: Mechanics of materials

Strength criterion Plasticizing due to the pressure of the mandrel against the walls of the hole



$$A_d = d \cdot t_1$$



$$A_d = d \cdot 2t$$

Stress due to the pressure of a single connector:

$$\sigma_d = \frac{P/n}{A_d}$$

Bearing surface:

$$A_d = d\sum_i t_i$$

Strength criterion for bearing

$$\frac{P}{d\sum_i t_i \cdot n} \leq \sigma_{allow.B}$$

d – connector diameter

n - number of force-transmitting connectors

$\sum_i t_i$– smaller of the total thickness of the plates pressed to one side of the connector

$\sigma_{allow.B}$- allowable bearing stress

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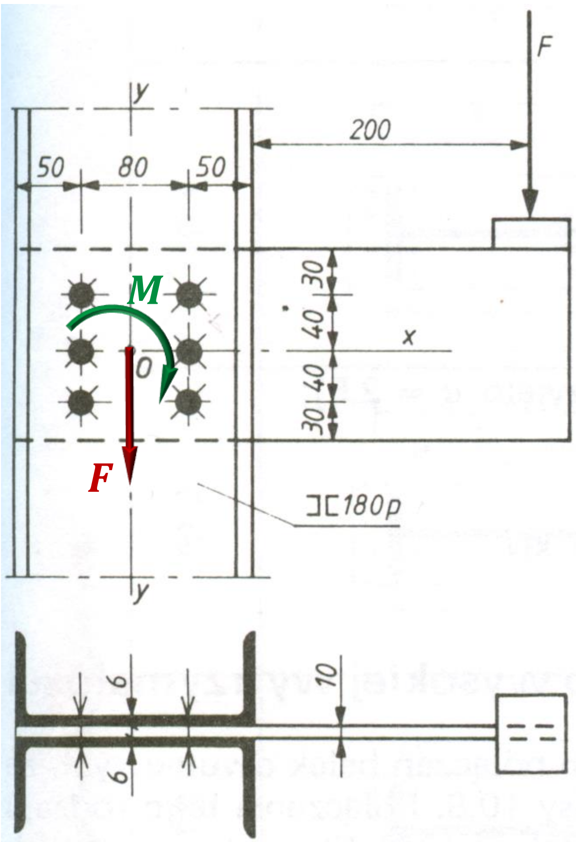
Calculation examples

Task 1

A beam was attached to the column by 6 bolts M12 of class 4.6. The connection is loaded by a force F = 15 kN according to the presented scheme. Check the strength criteria if allowable stresses for shear and bearing amount to $\sigma_{allow.S}$=180 MPa and $\sigma_{allow.B}$=537 MPa.



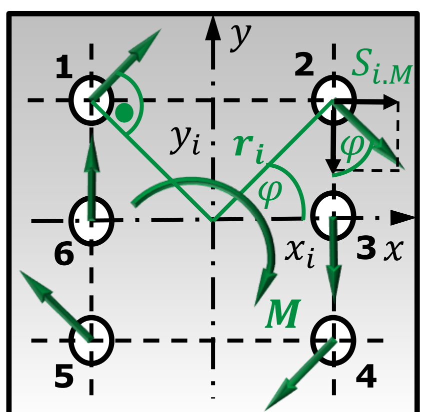
Force acting on a single bolt due to **loading of the vertical force *F***

$$\boldsymbol{S_{i.F}} = \frac{\boldsymbol{-F}}{\boldsymbol{n}} \quad n \text{ – number of connectors}$$

Assumption:

The force acting on a single connector as a result of the moment load is perpendicular to the position radius of the connector and proportional to its length: $\boldsymbol{S_{i.M} = k \cdot r_i}$

$$M = k \sum_i r_i^2 \Rightarrow k = \frac{M}{\sum_i (x_i^2 + y_i^2)} \quad \Rightarrow \quad S_{i.M} = \frac{M r_i}{\sum_i (x_i^2 + y_i^2)}$$



$$S_{i.M.x} = S_{i.M} \sin\varphi = \frac{y_i}{r_i} S_{i.M}$$

$$|S_{i.M.y}| = S_{i.M} \cos\varphi = \frac{x_i}{r_i} S_{i.M}$$

$$\boldsymbol{S_{i.M.x}} = \frac{\boldsymbol{M y_i}}{\sum_i (\boldsymbol{x_i^2 + y_i^2})} \qquad \boldsymbol{S_{i.M.y}} = \frac{\boldsymbol{-M x_i}}{\sum_i (\boldsymbol{x_i^2 + y_i^2})}$$

$$\boldsymbol{S_i} = \sqrt{\boldsymbol{S_{i.M.x}^2} + (\boldsymbol{S_{i.M.y}} + \boldsymbol{S_{i.F}})^2}$$

Calculation example

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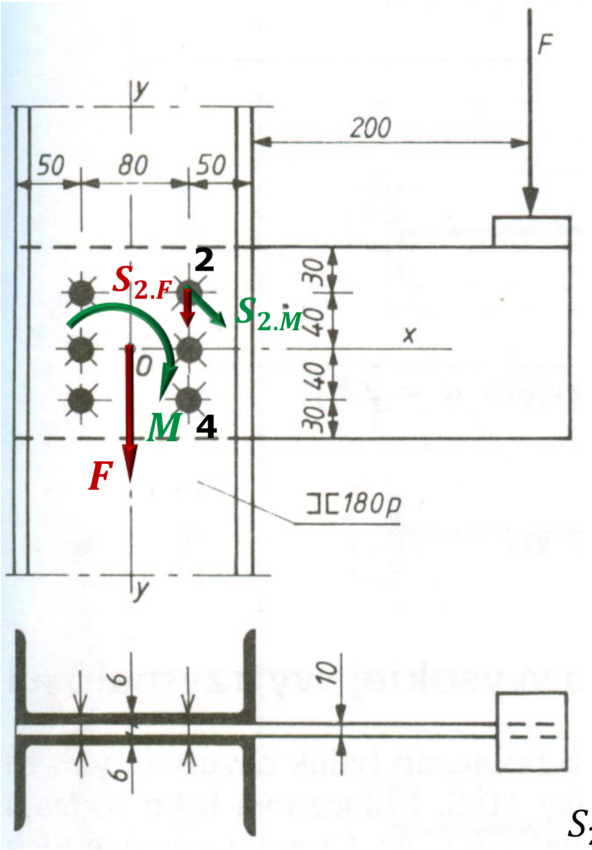


Force acting on a single bolt due to **loading of the vertical force *F***

$$S_{i.F} = \frac{-15}{6} = -2.5 \text{ kN}$$

Force acting on a single bolt due to **loading of the moment *M***

$$M = 15 \cdot 0.29 = 4.35 \text{ kNm}$$

$$S_{2.M.x} = \frac{4.35 \text{ kNm} \cdot 4 \cdot 10^{-2}\text{m}}{(6 \cdot 4^2 + 4 \cdot 4^2) \cdot 10^{-4}\text{m}^2} = 10.9 \text{ kN} = -S_{2.M.y}$$

$$S_2 = \sqrt{10.9^2 + (-10.9 - 2.5)^2} = 17.3 \text{ kN}$$

Strength criterion: shearing

$$\frac{S_2}{2\frac{\pi d^2}{4}} \leq \sigma_{allow.S} \Leftrightarrow 76.5 \text{ MPa} < 180 \text{ MPa} \quad \checkmark$$

Strength criterion: bearing

$$\frac{S_2}{t \cdot d} \leq \sigma_{allow.B} \Leftrightarrow \frac{17.3 \text{ kN}}{1.2 \cdot 1.0 \cdot 10^{-4} \text{ m}^2} \leq 537.5 \text{ MPa} \Leftrightarrow 144.2 \text{ MPa} < 537 \text{ MPa} \quad \checkmark$$

Strength calculations
Shearing of fillet welds

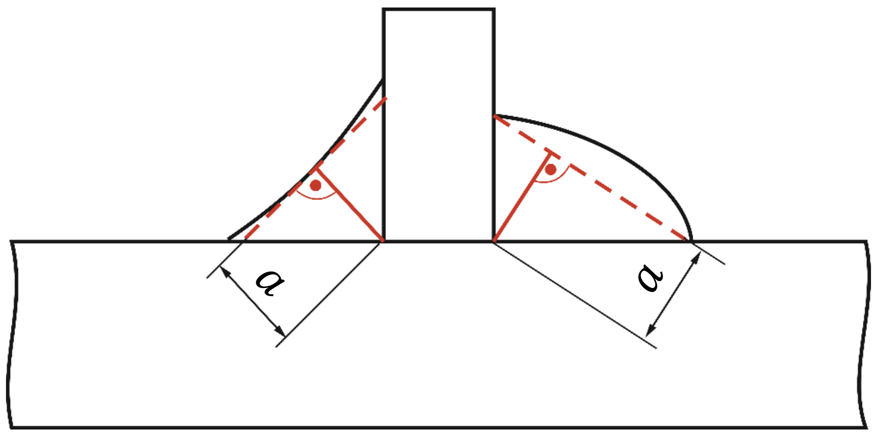
Shear stress:

$$\tau = \frac{P}{A_t}$$

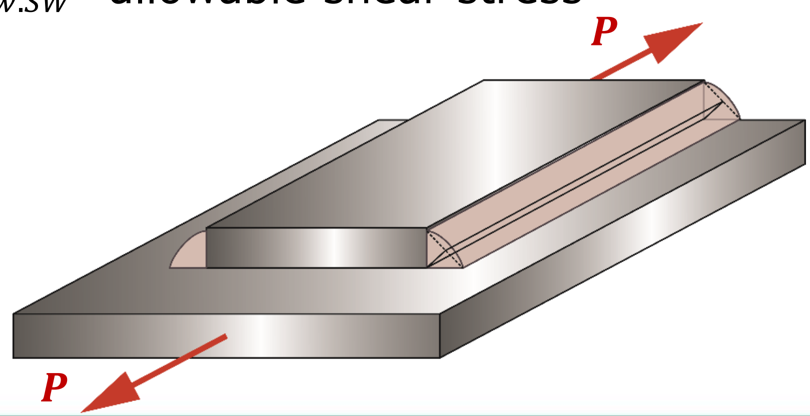
Strength criterion for weld shearing:

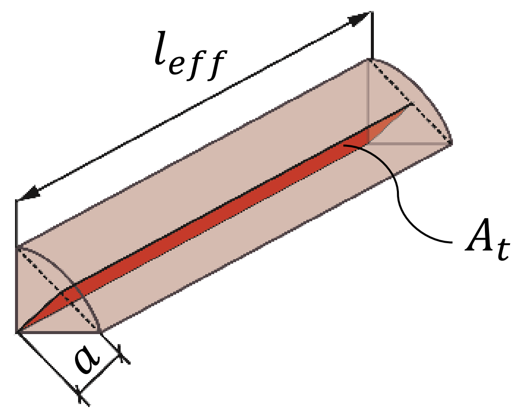
$$\frac{P}{a \cdot l_{eff}} \leq \sigma_{allow.SW}$$

A_t - area of shear surface
a – weld thickness
l_{eff} - effective weld length
$\sigma_{allow.SW}$- allowable shear stress

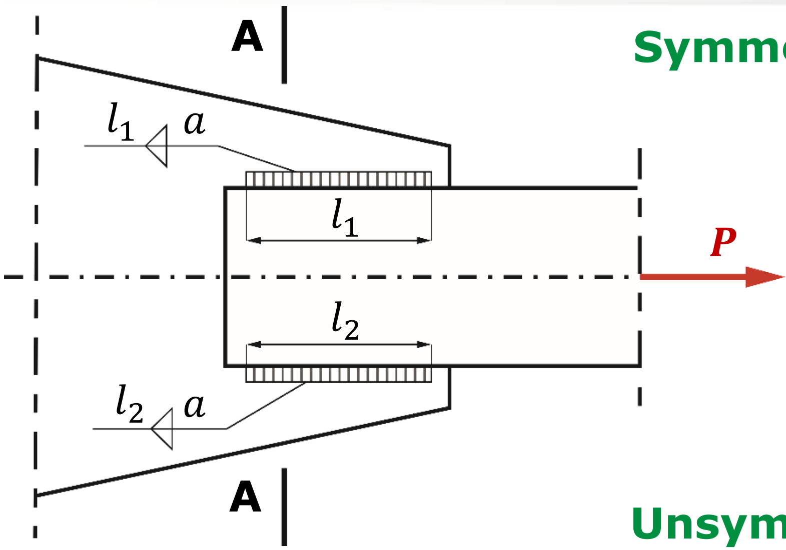


Standard: Design of steel structures - Part 1-8: Design of joints EN 1993-1-8:2024

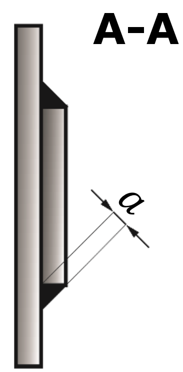




Rules for selecting the length of longitudinal fillet welds



Symmetrical joint



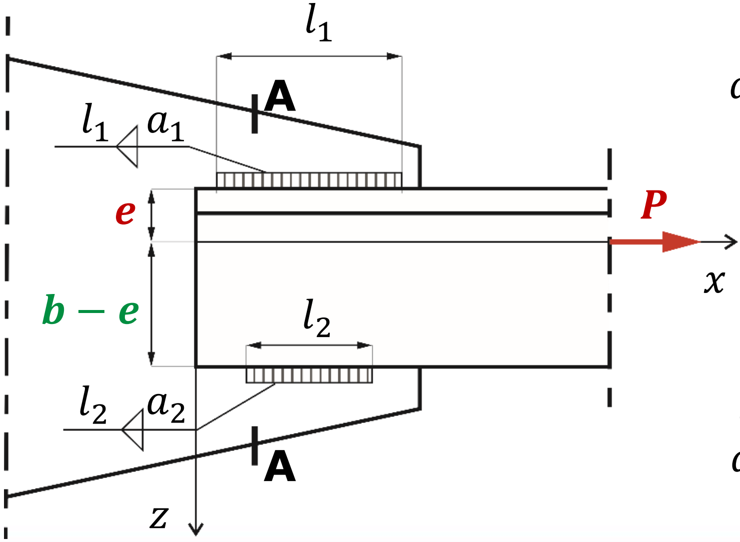
$$l_1 = l_2 = l$$

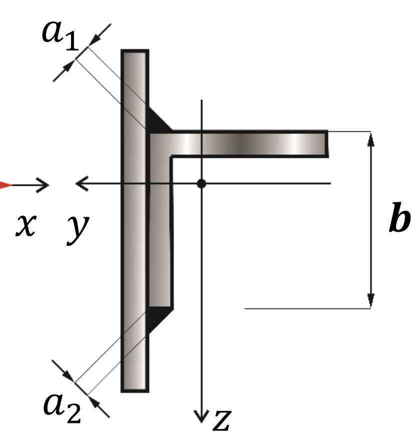
$$l_c = l_1 + l_2$$

$$l_c = 2l$$

$$\frac{P}{a \cdot l_c} \le \sigma_{allow.SW}$$

Unsymmetrical joint





$$z_c = \frac{S_x}{A} = 0$$

$$\frac{-l_1 a_1 e + a_2 l_2 (b-e)}{l_1 a_1 + a_2 l_2} = 0$$

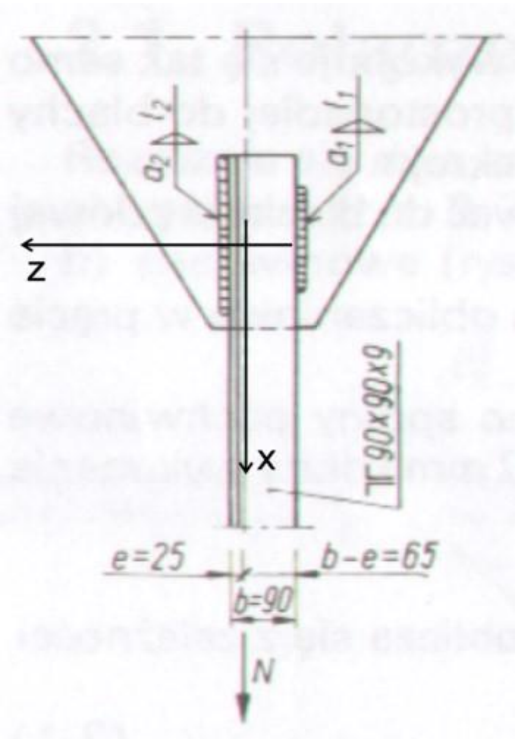
if: $a_1 = a_2$

$$l_1 e = l_2 (b-e)$$

$$l_c = l_1 + l_2$$

Task 2

Two angles 90x90x9 should be attached with fillet welds with a thickness of a = 5mm, arranged in a way ensuring axial force transfer N = 660 kN. Design the weld length if the allowable shear stress for the weld $\sigma_{allow.SW}$ = 172 MPa.



Total weld length: $l = 2(l_1 + l_2)$

Shear condition: $\dfrac{N}{l \cdot a} \leq \sigma_{allow.SW}$, $l \geq \dfrac{N}{a \cdot \sigma_{allow.SW}}$,

$l \geq 767\ \text{mm}$ Assum.: $l = 770\ \text{mm}$

Condition of the axial force transfer:

$$2ael_2 = 2a(b-e)l_1 \implies l_2 = \frac{(b-e)}{e}l_1, \quad l_2 = \frac{13}{5}l_1$$

$$l = 2\left(l_1 + \frac{13}{5}l_1\right), \quad l_1 = \frac{5}{36}l, \quad l_1 = 107\ \text{mm}$$

$$l_2 = \frac{13}{36}l, \quad l_2 = 278\ \text{mm}$$